integrales por partes ...

1) $\int x \cdot \ln x \, dx =$

2) $\int x^2 \cdot e^x \, dx =$

3) $\int x \cdot \operatorname{sen} x \, dx =$

4) $\int x \cdot e^x \, dx =$

5) $\int x^2 \cdot \ln x \, dx =$

6) $\int x \cdot \cos x \, dx =$

7) $\int \arctan x \, dx =$

8) $\int e^x \cdot \cos x \, dx =$

9) $\int \operatorname{arcsen} x \, dx =$

10) $\int \sqrt{x} \cdot \ln x \, dx =$

11) $\int \frac{x}{\cos^2 x} \, dx =$

$$\int u \, dv = uv - \int v \, du.$$

Solución ...

1) $\int x \cdot \ln x \, dx = \ln x \cdot \frac{x^2}{2} - \int \frac{1}{x} \cdot \frac{x^2}{2} \, dx = \frac{x^2 \cdot \ln x}{2} - \frac{1}{2} \int x \, dx =$

A
L $\longrightarrow \ln x$
P $\longrightarrow x$
E
S

$u = \ln x \rightarrow du = \frac{1}{x} \cdot dx$

$dv = x \, dx \longrightarrow v = \frac{x^2}{2}$

$$= \frac{x^2 \cdot \ln x}{2} - \frac{x^2}{4} + k$$

2) $\int x^2 \cdot e^x \, dx = x^2 \cdot e^x - \int 2x \cdot e^x \, dx = x^2 \cdot e^x - 2\int x \cdot e^x \, dx = [*]$

ALPES → x^2, e^x

$u = x^2 \longrightarrow du = 2x\,dx$

$dv = e^x\,dx \longrightarrow v = e^x$

$u = x \longrightarrow du = dx$

$dv = e^x\,dx \longrightarrow v = e^x$

$[*] = x^2 \cdot e^x - 2 \cdot \left[x \cdot e^x - \int e^x \, dx \right] = x^2 \cdot e^x - 2x \cdot e^x + 2e^x + K$

$e^x[x^2 - 2x + 2] + K$

3) $\int x \cdot \operatorname{sen} x \, dx = -x \cdot \cos x - \int -\cos x \, dx = -x \cdot \cos x + \int \cos x \, dx$

$= -x \cdot \cos x + \operatorname{sen} x + K$

ALPES → x, $\operatorname{sen} x$

$u = x \longrightarrow du = dx$

$dv = \operatorname{sen} x\,dx \longrightarrow v = -\cos x$

4) $\int x \cdot e^x \, dx = x \cdot e^x - \int e^x \, dx = x \cdot e^x - e^x + K = e^x[x - 1] + K$

ALPES → x, e^x

$u = x \longrightarrow du = dx$

$dv = e^x\,dx \longrightarrow v = e^x$

5) $\int x^2 \cdot \ln x \, dx = \frac{x^3 \cdot \ln x}{3} - \int \frac{x^3}{3} \cdot \frac{1}{x} \, dx = \frac{x^3 \cdot \ln x}{3} - \frac{1}{3}\int x^2 \, dx =$

ALPES → $\ln x$, x^2

$u = \ln x \longrightarrow du = \frac{1}{x}\,dx$

$dv = x^2\,dx \longrightarrow v = \frac{x^3}{3}$

$= \frac{x^3 \cdot \ln x}{3} - \frac{x^3}{9} + K$

6) $\int x \cdot \cos x \, dx = x \cdot \operatorname{sen} x - \int \operatorname{sen} x \, dx = x \cdot \operatorname{sen} x + \cos x + K$

ALPES

P → x

S → $\cos x$

$u = x \longrightarrow du = dx$

$dv = \cos x \, dx \longrightarrow v = \operatorname{sen} x$

7) $\int \arctan x \, dx = x \cdot \arctan x - \int \frac{x}{1+x^2} \, dx = x \cdot \arctan x - \frac{1}{2} \int \frac{2x}{1+x^2} \, dx =$

$u = \arctan x \longrightarrow du = \frac{1}{1+x^2} \, dx$

$dv = dx \longrightarrow v = x$

$= x \cdot \arctan x - \frac{1}{2} \ln |1+x^2| + K$

8) $\int e^x \cdot \cos x \, dx = e^x \cdot \operatorname{sen} x - \int e^x \cdot \operatorname{sen} x \, dx =$

$u = e^x \longrightarrow du = e^x \, dx$

$dv = \cos x \, dx \longrightarrow v = \operatorname{sen} x$

$u = e^x \longrightarrow du = e^x \, dx$

$dv = \operatorname{sen} x \, dx \longrightarrow v = -\cos x$

$= e^x \cdot \operatorname{sen} x - \left[e^x (-\cos x) - \int -\cos x \cdot e^x \, dx \right] =$

$= e^x \cdot \operatorname{sen} x + e^x \cdot \cos x - \int \cos x \, e^x \, dx$

$\int e^x \cdot \cos x \, dx = e^x \cdot \operatorname{sen} x + e^x \cdot \cos x - \int \cos x \cdot e^x \, dx$

$I + I = e^x \cdot \operatorname{sen} x + e^x \cdot \cos x$

$2I = e^x \cdot \operatorname{sen} x + e^x \cdot \cos x$

$I = \frac{e^x \cdot \operatorname{sen} x + e^x \cdot \cos x}{2} + K$

9) $\int \operatorname{arcsen} x \, dx = x \cdot \operatorname{arcsen} x - \int \frac{x}{\sqrt{1-x^2}} dx =$

$u = \operatorname{arcsen} x \longrightarrow du = \frac{1}{\sqrt{1-x^2}} dx$

$dv = dx \longrightarrow v = x$

$= x \cdot \operatorname{arcsen} x - \frac{1}{2} \int 2x (1-x^2)^{-1/2} dx =$

$= x \cdot \operatorname{arcsen} x - \frac{1}{2} \cdot \frac{\sqrt{1-x^2}}{\frac{1}{2}} + K = \boxed{x \cdot \operatorname{arcsen} x - \sqrt{1-x^2} + K}$

10) $\int \sqrt{x} \cdot \ln x \, dx = - \int \frac{2}{3} \sqrt{x^3} \cdot \frac{1}{x} dx =$

$u = \ln x \longrightarrow du = \frac{1}{x} dx$

$dv = \sqrt{x} \, dx \longrightarrow v = \frac{x^{\frac{3}{2}}}{\frac{3}{2}} = \frac{2}{3} \cdot \sqrt{x^3}$

$= \frac{2 \ln x \cdot \sqrt{x^3}}{3} - \frac{2}{3} \int \frac{x^{3/2}}{x} dx =$

$= \frac{2 \ln x \cdot \sqrt{x^3}}{3} - \frac{2}{3} \int x^{1/2} dx = \frac{3 \cdot \ln x \cdot \sqrt[3]{x^2}}{2} - \frac{2}{3} \cdot \frac{x^{3/2}}{\frac{3}{2}} + K =$

$= \boxed{\frac{2 \ln x \cdot \sqrt{x^3}}{3} - \frac{4}{9} \sqrt{x^3} + K}$

11) $\int \frac{x}{\cos^2 x} dx = x \cdot \tan x - \int \tan dx = x \cdot \tan x - \int \frac{\operatorname{sen} x}{\cos x} dx =$

$u = x \longrightarrow du = dx$

$dv = \frac{1}{\cos^2 x} dx \longrightarrow v = \tan x$

$= x \cdot \tan x + \int \frac{-\operatorname{sen} x}{\cos x} dx =$

$= \boxed{x \cdot \tan x + \ln |\cos x| + K}$