Integrales ... integrales inmediatas.

Este documento te servirá de iniciación en el mundo de las integrales ... Comenzamos ...

- $$\int [f(x)]^n \cdot f'(x)\, dx = \frac{[f(x)]^{n+1}}{n+1} + K$$

1) $\int -5x\,(3x^2-7)^{10}\,dx =$

2) $\int (3x-1)\cdot(3x^2-2x+1)^8\,dx =$

3) $\int x\sqrt{x^2-1}\,dx$

4) $\int \frac{x^2}{(x^3-5)^4}\,dx$

5) $\int \frac{5x}{\sqrt{3-x^2}}\,dx =$

6) $\int \frac{2x+5}{\sqrt[3]{x^2+5x}}\,dx =$

7) $\int \cos x \cdot [\sin x]^6\,dx =$

Soluciones ...

1) $\int -5x\,(3x^2-7)^{10}\,dx = -5\int x\,(3x^2-7)^{10}\,dx = \frac{-5}{6}\int 6x(3x^2-7)^{10}\,dx =$

$$= \boxed{\frac{-5}{6}\cdot\frac{(3x^2-7)^{11}}{11} + K}$$

2) $\int (3x-1)\cdot(3x^2-2x+1)^8\,dx = \frac{1}{2}\int 2(3x-1)\cdot(3x^2-2x+1)^8\,dx =$

$$= \boxed{\frac{1}{2}\,\frac{(3x^2-2x+1)^9}{9} + K}$$

3) $\int x\sqrt{x^2-1}\,dx = \int x\cdot(x^2-1)^{1/2}\,dx = \frac{1}{2}\int 2x\cdot(x^2-1)^{1/2}\,dx =$

$= \frac{1}{2}\,\frac{(x^2-1)^{3/2}}{\frac{3}{2}} + K = \boxed{\frac{1}{3}\sqrt{(x^2-1)^3} + K}$

4) $\int \frac{x^2}{(x^3-5)^4}\,dx = \int x^2(x^3-5)^{-4}\,dx = \frac{1}{3}\int 3x^2(x^3-5)^{-4}\,dx =$

$= \frac{1}{3}\,\frac{(x^3-5)^{-3}}{-3} + K = \boxed{\frac{1}{-9(x^3-5)^3} + K}$

5) $\int \frac{5x}{\sqrt{3-x^2}}\,dx = \int 5x\cdot(3-x^2)^{-1/2}\,dx = -\frac{5}{2}\int -2x\cdot(3-x^2)^{-1/2}\,dx = \boxed{-5(3-x^2)^{1/2}+K}$

6) $\int \frac{2x+5}{\sqrt[3]{x^2+5x}}\,dx = \int (2x+5)\cdot(x^2+5x)^{-1/3}\,dx = \boxed{\frac{3\sqrt[3]{[x^2+5x]^2}}{2}+K}$

7) $\int \cos x\cdot[\sin x]^6\,dx = \boxed{\frac{[\sin x]^7}{7}+K}$

- $\int \frac{f'(x)}{f(x)}\,dx = \ln|f(x)| + K.$

1) $\int \frac{1}{5x-1}\,dx =$

2) $\int \frac{2x}{x^2+3}\,dx =$

3) $\int \frac{x-3}{x^2-6x+3}\,dx =$

4) $\int \frac{x^2}{1-x^3}\,dx =$

5) $\int \frac{x^2+6x-2}{x^3+9x^2-6x+7}\,dx =$

6) $\int \tan x \, dx =$

7) $\int \cotan x \, dx =$

8) $\int \frac{\sin x}{1-\cos x} \, dx =$

9) $\int \frac{1}{\cos^2 x \cdot \tan x} \, dx =$

10) $\int \frac{2e^x}{5e^x+3} \, dx =$

11) $\int \frac{1}{(1+x^2)\arctan x} \, dx =$

12) $\int \frac{1}{x(1-\ln x)} \, dx =$

13) $\int \frac{2x-1}{x-2} \, dx =$

soluciones ...

1) $\int \frac{1}{5x-1} \, dx = \frac{1}{5}\int \frac{5}{5x-1} \, dx = \boxed{\frac{1}{5} \cdot \ln|5x-1| + K}$

2) $\int \frac{2x}{x^2+3} \, dx = \boxed{\ln|x^2+3| + K}$

3) $\int \frac{x-3}{x^2-6x+3} \, dx = \frac{1}{2}\int \frac{2(x-3)}{x^2-6x+3} \, dx = \frac{1}{2}\int \frac{2x-6}{x^2-6x+3} \, dx = \boxed{\frac{1}{2} \ln|x^2-6x+3| + K}$

4) $\int \frac{x^2}{1-x^3} \, dx = \frac{1}{-3}\int \frac{-3x^2}{1-x^3} \, dx = \boxed{-\frac{1}{3} \ln|1-x^3| + K}$

5) $\int \frac{x^2+6x-2}{x^3+9x^2-6x+7}\,dx = \frac{1}{3}\int \frac{3(x^2+6x-2)}{x^3+9x^2-6x+7}\,dx = \frac{1}{3}\int \frac{3x^2+18x-6}{x^3+9x^2-6x+7}\,dx$

$= \boxed{\frac{1}{3}\ln|x^3+9x^2-6x+7|+K}$

6) $\int \tan x\,dx = \int \frac{\sin x}{\cos x}\,dx = -\int \frac{-\sin x}{\cos x}\,dx = \boxed{-\ln|\cos x|+K}$

$D(\cos x) = -\sin x$

7) $\int \cot x\,dx = \int \frac{\cos x}{\sin x}\,dx = \boxed{\ln|\sin x|+K}$

$D(\sin x) = \cos x$

8) $\int \frac{\sin x}{1-\cos x}\,dx = \boxed{\ln|1-\cos x|+K}$

$D(1-\cos x) = \sin x$

$D(\tan x) = \frac{1}{\cos^2 x}$

9) $\int \frac{1}{\cos^2 x \cdot \tan x}\,dx = \int \frac{\frac{1}{\cos^2 x}}{\tan x}\,dx = \boxed{\ln|\tan x|+K}$

10) $\int \frac{2e^x}{5e^x+3}\,dx = 2\int \frac{e^x}{5e^x+3}\,dx = \frac{2}{5}\int \frac{5e^x}{5e^x+3}\,dx = \boxed{\frac{2}{5}\ln|5e^x+3|+K}$

11) $\int \frac{1}{(1+x^2)\arctan x}\,dx = \int \frac{\frac{1}{1+x^2}}{\arctan x}\,dx = \boxed{\ln|\arctan x|+K}$

12) $\int \frac{1}{x(1-\ln x)}\,dx = \int \frac{\frac{1}{x}}{(1-\ln x)}\,dx = -\int \frac{-\frac{1}{x}}{(1-\ln x)}\,dx = \boxed{-\ln|1-\ln x|+K}$

13) $\int \frac{2x-1}{x-2}\,dx = \int 2 + \frac{3}{x-2}\,dx = \boxed{2x+3\ln|x-2|+K}$

Cuando el grado de arriba y el grado de abajo sean iguales...

$$\begin{array}{r|l} P(x) & Q(x) \\ \hline \downarrow & Z(x) \\ H(x) & \end{array}$$

$$\begin{array}{r|l} 2x-1 & x-2 \\ \hline -2x+4 & 2 \\ \hline 3 & \end{array}$$

$$\int \frac{P(x)}{Q(x)}\,dx = \int Z(x) + \frac{H(x)}{Q(x)}\,dx$$

$$\bullet \int e^{f(x)} \cdot f'(x)\, dx = e^{f(x)} + K.$$

1) $\int 2 e^{x}\, dx =$

2) $\int 3\, e^{5x-2}\, dx =$

3) $\int 5 e^{-x}\, dx =$

4) $\int e^{3-2x}\, dx =$

5) $\int 2x \cdot e^{x^2} dx =$

6) $\int 3x \cdot e^{x^2-7} dx =$

7) $\int x \cdot e^{1-x^2} dx =$

8) $\int (x-2) \cdot e^{x^2-4x}\, dx =$

9) $\int \frac{1}{e^{5x}}\, dx =$

10) $\int \frac{1}{e^{3-2x}}\, dx =$

11) $\int x^3 e^{x^4+5} dx =$

12) $\int \frac{e^{x}}{e^{-x}}\, dx =$

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Soluciones ...

1) $\int 2 e^{x}\, dx = 2\int e^{x} dx = \boxed{2 e^{x} + K}$

2) $\int 3\, e^{5x-2}\, dx = 3\int e^{5x-2} dx = \frac{3}{5}\int 5 \cdot e^{5x-2} dx = \boxed{\frac{3}{5} \cdot e^{5x-2} + K}$

3) $\int 5e^{-x}\,dx = 5\int e^{-x}\,dx = -5\int -e^{-x}\,dx = \boxed{-5\cdot e^{-x}+K}$

4) $\int e^{3-2x}\,dx = \frac{1}{-2}\int -2\cdot e^{3-2x}\,dx = \boxed{-\frac{1}{2}e^{3-2x}+K}$

5) $\int 2x\cdot e^{x^2}\,dx = \boxed{e^{x^2}+K}$

6) $\int 3x\cdot e^{x^2-7}\,dx = 3\int x\cdot e^{x^2-7}\,dx = \frac{3}{2}\int 2x\cdot e^{x^2-7}\,dx = \boxed{\frac{3}{2}e^{x^2-7}+K}$

7) $\int x\cdot e^{1-x^2}\,dx = -\frac{1}{2}\int -2x\cdot e^{1-x^2}\,dx = \boxed{-\frac{1}{2}e^{1-x^2}+K}$

8) $\int (x-2)\cdot e^{x^2-4x}\,dx = \frac{1}{2}\int (2x-4)\cdot e^{x^2-4x}\,dx = \boxed{\frac{1}{2}e^{x^2-4x}+K}$

9) $\int \frac{1}{e^{5x}}\,dx = \int e^{-5x}\,dx = -\frac{1}{5}\int -5\cdot e^{-5x}\,dx = \boxed{-\frac{1}{5}\cdot e^{-5x}+K}$

10) $\int \frac{1}{e^{3-2x}}\,dx = \int e^{-3+2x}\,dx = \frac{1}{2}\int 2\cdot e^{-3+2x}\,dx = \boxed{\frac{1}{2}e^{-3+2x}+K}$

11) $\int x^3 e^{x^4+5}\,dx = \frac{1}{4}\int 4x^3\cdot e^{x^4+5}\,dx = \boxed{\frac{1}{4}\cdot e^{x^4+5}+K}$

12) $\int \frac{e^x}{e^{-x}}\,dx = \int e^{x-(-x)}\,dx = \int e^{2x}\,dx = \frac{1}{2}\int 2\cdot e^{2x}\,dx = \boxed{\frac{1}{2}e^{2x}+K}$

Recuerda:

$$\frac{A^m}{A^n} = A^{m-n}$$

• $$\int a^{f(x)}\cdot f'(x)\,dx = \frac{a^{f(x)}}{\ln a}+K$$

1) $\int 2\cdot 3^x\,dx =$

2) $\int 3\cdot 2^{5x-2}\,dx =$

3) $\int 5\cdot 2^{-x}\,dx =$

4) $\int 5^{3-2x}\,dx =$

5) $\int 3x \cdot 10^{x^2-9} =$

6) $\int \frac{1}{2^{5x}}\, dx =$

7) $\int \frac{1}{2^{3-2x}}\, dx =$

8) $\int \frac{x^3 \cdot 3^{x^4+5}}{\ln 3}\, dx =$

9) $\int \frac{2^x}{2^{-x}}\, dx =$

10) $\int \frac{5^{3x}}{5^{-x}}\, dx =$

11) $\int \frac{(2x+1)\cdot 3^{x^2}}{3^{-x}}\, dx =$

Soluciones ...

1) $\int 2 \cdot 3^x\, dx = 2\int 3^x\, dx = \boxed{2 \cdot \frac{3^x}{\ln 3} + K}$

2) $\int 3 \cdot 2^{5x-2}\, dx = 3\int 2^{5x-2}\, dx = \frac{3}{5}\int 5 \cdot 2^{5x-2}\, dx = \boxed{\frac{3\, 2^{5x-2}}{5 \cdot \ln 2} + K}$

3) $\int 5 \cdot 2^{-x}\, dx = 5 \cdot \int 2^{-x}\, dx = -5\int -2^{-x}\, dx = \boxed{-5 \cdot \frac{2^{-x}}{\ln 2} + K}$

4) $\int 5^{3-2x}\, dx = -\frac{1}{2}\int -2 \cdot 5^{3-2x}\, dx = \boxed{-\frac{1}{2} \cdot \frac{5^{3-2x}}{\ln 5} + K}$

5) $\int 3x \cdot 10^{x^2-9} = 3\int x \cdot 10^{x^2-9}\, dx = \frac{3}{2}\int 2x \cdot 10^{x^2-9}\, dx = \boxed{\frac{3}{2} \cdot \frac{10^{x^2-9}}{\ln 10} + K}$

6) $\int \frac{1}{2^{5x}}\, dx = \int 2^{-5x}\, dx = -\frac{1}{5}\int -5 \cdot 2^{-5x}\, dx = \boxed{-\frac{1}{5} \cdot \frac{2^{-5x}}{\ln 2} + K}$

7) $\int \frac{1}{2^{3-2x}}\, dx = \int 2^{-3+2x}\, dx = \frac{1}{2}\int 2 \cdot 2^{-3+2x}\, dx = \boxed{\frac{1}{2} \cdot \frac{2^{-3+2x}}{\ln 2} + K}$

8) $\int \frac{x^3 \cdot 3^{x^4+5}}{\ln 3}\, dx = \frac{1}{\ln 3}\int x^3 \cdot 3^{x^4+5}\, dx = \frac{1}{4\ln 3} \cdot \int 4x^3 \cdot 3^{x^4+5}\, dx =$

$\boxed{= \frac{1}{4 \cdot \ln 3} \cdot \frac{3^{x^4+5}}{\ln 3} + K}$.

9) $\int \frac{2^x}{2^{-x}}\, dx = \int 2^x \cdot 2^x\, dx = \int 2^{2x}\, dx = \frac{1}{2}\int 2 \cdot 2^{2x}\, dx =$

$$= \boxed{\frac{1}{2} \cdot \frac{2^{2x}}{\ln 2} + K}$$

10) $\int \frac{5^{3x}}{5^{-x}}\, dx = \int 5^{3x} \cdot 5^x\, dx = \int 5^{4x}\, dx = \frac{1}{4}\int 4 \cdot 5^{4x}\, dx = \boxed{\frac{1}{4} \cdot \frac{5^{4x}}{\ln 5} + K}$

11) $\int \frac{(2x+1) \cdot 3^{x^2}}{3^{-x}}\, dx = \int (2x+1) \cdot 3^{x^2+x}\, dx = \boxed{\frac{3^{x^2+x}}{\ln 3} + K}$

- $\int \sin f(x) \cdot f'(x)\, dx = -\cos f(x) + K$
- $\int \cos f(x) \cdot f'(x)\, dx = \operatorname{sen} f(x) + K$
- $\int \frac{f'(x)}{\cos^2 f(x)}\, dx = \int \left(1 + \tan^2 f(x)\right) \cdot f'(x)\, dx = \tan f(x) + K$

1) $\int \cos(2x)\, dx =$

2) $\int \cos(3x-1)\, dx =$

3) $\int \sin(5x)\, dx =$

4) $\int \sin(1-3x)\, dx =$

5) $\int 2 \cdot \cos(3x)\, dx =$

6) $\int 3 \cdot \sin(2x)\, dx =$

7) $\int x^4 \cdot \cos(x^5)\, dx =$

8) $\int \frac{x^2}{\cos^2 x^3}\, dx =$

9) $\int \left(1+\tan^2(x^2)\right) \cdot x \, dx =$

soluciones ...

1) $\int \cos(2x)\, dx = \frac{1}{2}\int 2\cdot\cos(2x)\, dx = \boxed{\frac{1}{2}\sin(2x)+K}$

2) $\int \cos(3x-1)\, dx = \frac{1}{3}\int 3\cdot\cos(3x-1)\, dx = \boxed{\frac{1}{3}\operatorname{sen}(3x-1)+K}$

3) $\int \sin(5x)\, dx = \frac{1}{5}\int 5\cdot\sin(5x)\, dx = \boxed{-\frac{1}{5}\cdot\cos(5x)+K}$

4) $\int \sin(1-3x)\, dx = -\frac{1}{3}\int -3\sin(1-3x)\, dx = \boxed{\frac{1}{3}\cos(1-3x)+K}$

5) $\int 2\cdot\cos(3x)\, dx = 2\cdot\int\cos(3x)\, dx = \frac{1}{3}\cdot 2\cdot\int 3\cdot\cos(3x)\, dx =$

$= \boxed{\frac{2}{3}\cdot\sin(3x)+K}$

6) $\int 3\cdot\sin(2x)\, dx = 3\int\sin(2x)\, dx = \frac{3}{2}\int 2\cdot\sin(2x)\, dx =$

$\boxed{= -\frac{3}{2}\cdot\cos(2x)\, dx}$

7) $\int x^4\cdot\cos(x^5)\, dx = \frac{1}{5}\int 5x^4\cdot\cos(x^5)\, dx = \boxed{+\frac{1}{5}\operatorname{sen}(x^5)+K}$

8) $\int \frac{x^2}{\cos^2 x^3}\, dx = \frac{1}{3}\int\frac{3x^2}{\cos^2 x^3}\, dx = \boxed{\frac{1}{3}\tan x^3 + K}$

9) $\int \left(1+\tan^2(x^2)\right)\cdot x\, dx = \frac{1}{2}\int\left[1+\tan^2(x^2)\right]\cdot 2x\, dx =$

$\boxed{= \frac{1}{2}\cdot\tan x^2 + K}$

- $$\int \frac{f'(x)}{\sqrt{1-[f(x)]^2}}\,dx = \arcsin f(x) + K$$

- $$\int \frac{f'(x)}{1+[f(x)]^2}\,dx = \arctan f(x) + K.$$

1) $\int \frac{3}{1+x^2}\,dx =$

2) $\int \frac{3x}{1+x^4}\,dx =$

3) $\int \frac{e^x}{1+e^{2x}}\,dx =$

4) $\int \frac{1}{x(1+[\ln x]^2)}\,dx =$

5) $\int \frac{\sin x}{1+\cos^2 x}\,dx =$

6) $\int \frac{\cos x}{1+\sin^2 x}\,dx =$

7) $\int \frac{3}{1+9x^2}\,dx$

8) $\int \frac{7}{\sqrt{1-x^2}}\,dx =$

9) $\int \frac{2x}{\sqrt{1-x^4}}\,dx =$

10) $\int \frac{e^x}{\sqrt{1-e^{2x}}}\,dx =$

11) $\int \frac{\cos x}{\sqrt{1-\operatorname{sen}^2 x}}\,dx =$

soluciones ...

1) $\int \frac{3}{1+x^2}\,dx = 3\int \frac{1}{1+x^2}\,dx = \boxed{3 \cdot \arctan x + K}$

2) $\int \frac{3x}{1+x^4}\,dx = \int \frac{3x}{1+[x^2]^2}\,dx = \frac{3}{2}\int \frac{2x}{1+[x^2]^2}\,dx = \boxed{\frac{3}{2}\arctan x^2 + K}$

3) $\int \frac{e^x}{1+e^{2x}}\,dx = \int \frac{e^x}{1+[e^x]^2}\,dx = \boxed{\arctan e^x + K}$

4) $\int \frac{1}{x(1+[\ln x]^2)}\,dx = \int \frac{\frac{1}{x}}{1+[\ln x]^2}\,dx = \boxed{\arctan(\ln x) + K}$

5) $\int \frac{\sin x}{1+\cos^2 x}\,dx = -\int \frac{-\operatorname{sen} x}{1+\cos^2 x}\,dx = \boxed{-\arctan(\cos x) + K}$

6) $\int \frac{\cos x}{1+\sin^2 x}\,dx = \boxed{\arctan(\operatorname{sen} x) + K}$

7) $\int \frac{3}{1+9x^2}\,dx = \int \frac{3}{1+(3x)^2}\,dx = \boxed{\arctan 3x + K}$

8) $\int \frac{7}{\sqrt{1-x^2}}\,dx = 7\int \frac{1}{\sqrt{1-x^2}}\,dx = \boxed{7 \cdot \operatorname{arcsen} x + K}$

9) $\int \frac{2x}{\sqrt{1-x^4}}\,dx = \int \frac{2x}{\sqrt{1-(x^2)^2}}\,dx = \boxed{\operatorname{arcsen} x^2 + K}$

10) $\int \frac{e^x}{\sqrt{1-e^{2x}}}\,dx = \boxed{\operatorname{arcsen} e^x + K}$

11) $\int \frac{\cos x}{\sqrt{1-\operatorname{sen}^2 x}}\,dx = \boxed{\operatorname{arcsen}(\operatorname{sen} x) + K}$