C2 Academia

L'HOPITAL

Si encuentras algún fallo info@c2academia.com

$\lim\limits_{x\to 0}\frac{e^{x}-e^{-x}}{sen\ x}$	$\lim\limits_{x\to 0}\frac{e^{x}-e^{-x}}{sen\ x}=\frac{1-1}{0}=\frac{0}{0}$ (ind) $\lim\limits_{x\to 0}\frac{e^{x}-e^{-x}}{sen\ x}\xrightarrow{L'Hopital}\lim\limits_{x\to 0}\frac{e^{x}+e^{x}}{\cos x}=\frac{1+1}{1}=\frac{2}{1}=2$
$\lim\limits_{x\to\infty}\frac{x}{(\ln x)^3+2x}$	$\lim\limits_{x\to\infty}\frac{x}{(\ln x)^3+2x}=\frac{\infty}{\infty}$ (ind) $\lim\limits_{x\to\infty}\frac{x}{(\ln x)^3+2x}\xrightarrow{L'Hopital}\lim\limits_{x\to\infty}\frac{1}{3(\ln x)^2\cdot\frac{1}{x}+2}\to\frac{1}{0+2}=\frac{1}{2}$
$\lim\limits_{x\to 0}\left(\frac{1}{x}-\frac{1}{senx}\right)$	$\lim\limits_{x\to 0}\left(\frac{1}{x}-\frac{1}{sen\ x}\right)=\infty-\infty$ (ind) $\lim\limits_{x\to 0}\left(\frac{1}{x}-\frac{1}{sen\ x}\right)=\lim\limits_{x\to 0}\frac{sen\ x-x}{x\cdot sen\ x}=\frac{0-0}{0}=\frac{0}{0}$ (ind) $\lim\limits_{x\to 0}\frac{sen\ x-x}{x\cdot sen\ x}\xrightarrow{L'Hopital}\lim\limits_{x\to 0}\frac{\cos x-x}{sen\ x+x\cos x}=\frac{1}{0+0}=\frac{1}{0}$ (ind) (límites laterales)
$\lim\limits_{x\to\frac{\pi}{4}}(\tan x-1)\sec 2x$	$\lim\limits_{x\to\frac{\pi}{4}}(\tan x-1)\cdot\sec 2x=0\cdot\infty$ (ind) ¡Atención! $f(x)\cdot g(x)=\frac{f(x)}{\frac{1}{g(x)}}=\frac{g(x)}{\frac{1}{f(x)}}$ puedes utilizar cualquiera de las igualdades ... siempre haremos la más sencilla. En esta ocasión $\sec 2x=\frac{1}{\cos 2x}$ $\lim\limits_{x\to\frac{\pi}{4}}(\tan x-1)\cdot\sec 2x=\lim\limits_{x\to\frac{\pi}{4}}\frac{(\tan x-1)}{\cos 2x}=\frac{0}{0}$ (ind) $\lim\limits_{x\to\frac{\pi}{4}}\frac{(\tan x-1)}{\cos 2x}\xrightarrow{L'Hopital}\lim\limits_{x\to\frac{\pi}{4}}\frac{1+\tan^2 x}{-2\,sen\ 2x}=\frac{2}{-2}=-1$

¡¡Atención!!

$$f(x)\cdot g(x)=\frac{f(x)}{\frac{1}{g(x)}}=\frac{g(x)}{\frac{1}{f(x)}}$$

esta transformación la utilizamos cuando tenemos una indeterminación del tipo: $0\cdot\infty$

$\lim_{x\to0} x^{tanx}$	$\lim_{x\to 0} x^{\tan x} = 0^0$ (ind) $\ln \lim_{x\to0} x^{\tan x} = \ln L \longrightarrow \underbrace{\lim_{x\to 0} \tan x \cdot \ln x}_{(*)} = \ln L \longrightarrow 0 = \ln L$ $e^0 = L$ $\boxed{1 = L}$ (*) $\lim_{x\to0^+} \tan x \cdot \ln x = 0\cdot(-\infty)$ ind $\lim_{x\to0^+} \frac{\tan x}{\frac{1}{\ln x}} = \frac{0}{0}$ (ind) ... $\lim_{x\to0^+}\frac{\tan x}{\frac{1}{\ln x}} \xrightarrow{\text{L'Hopital}} \lim_{x\to0^+}\frac{1+\tan^2 x}{\frac{-1\cdot\frac1x}{(\ln x)^2}} = \lim_{x\to0^+}\frac{1+\tan^2 x}{\frac{-1}{x\cdot(\ln x)^2}} =$ $= \lim_{x\to0^+}\frac{[1+\tan^2x]\cdot x\cdot(\ln x)^2}{-1} = 0$
$\lim_{x\to0}(cotan\ x)^{sen\ x}$	$\lim_{x\to0}\left(\frac{1}{\tan x}\right)^{\operatorname{sen} x} = \infty^0$ (ind) $A = \lim_{x\to0}\left(\frac1{\tan x}\right)^{\operatorname{sen}x} \longrightarrow \ln A = \ln\lim_{x\to0}\left(\frac1{\tan x}\right)^{\operatorname{sen}x} \longrightarrow \ln A = \underbrace{\lim_{x\to0}\operatorname{sen}x\cdot\ln\left(\frac1{\tan x}\right)}_{(*)} \longrightarrow \ln A = 0$ $A = e^0$ $\boxed{A = 1}$ (*) $= 0\cdot\infty$ (ind) $\longrightarrow \lim_{x\to0}\frac{\ln\left(\frac1{\tan x}\right)}{\frac1{\operatorname{sen}x}} = \frac{\infty}{\infty}$ (ind) $\longrightarrow \lim_{x\to0}\frac{\ln\left(\frac1{\tan x}\right)}{\frac1{\operatorname{sen}x}} \xrightarrow{\text{L'Hopital}}$ $\longrightarrow \lim_{x\to0}\frac{\frac{\operatorname{cosec}^2x}{\operatorname{cotan}x}}{\frac{-\cos x}{\operatorname{sen}^2x}} = \lim_{x\to0}\frac{\operatorname{cosec}^2x\cdot\operatorname{sen}^2x}{-\operatorname{cotan}x\cdot\cos x} = \lim_{x\to0}\frac{\frac{1}{\operatorname{sen}^2x}\cdot\operatorname{sen}^2x}{\frac{-\cos^2x}{\operatorname{sen}x}} = \lim_{x\to0}\frac{1}{\frac{-\cos^2x}{\operatorname{sen}x}} = \lim_{x\to0}\frac{\operatorname{sen}x}{-\cos^2x} = 0$
$\lim_{x\to2}\left(\frac x2\right)^{\frac1{x-2}}$	$\lim_{x\to2}\left(\frac x2\right)^{\frac1{x-2}} = 1^\infty$ (ind) $A = \lim_{x\to2}\left(\frac x2\right)^{\frac1{x-2}} \longrightarrow \ln A = \ln \lim_{x\to2}\left(\frac x2\right)^{\frac1{x-2}} \longrightarrow \ln A = \underbrace{\lim_{x\to2}\left(\frac1{x-2}\right)\cdot\ln\left(\frac x2\right)}_{(*)} \longrightarrow \ln A = \frac12$ $A = e^{1/2}$ $\boxed{A=\sqrt e}$ (*) $\lim_{x\to2}\frac1{x-2}\cdot\ln\left(\frac x2\right) = \infty\cdot0 \longrightarrow \lim_{x\to2}\frac{\ln\left(\frac x2\right)}{x-2} = \frac00$ (ind) L'Hopital $\lim_{x\to2}\frac{\frac{1/2}{x/2}}{1} \longrightarrow \lim_{x\to2}\frac1x = \frac12$
$\lim_{x\to1}\frac{\ln x}{x-1}$	$\lim_{x\to1}\frac{\ln x}{x-1} = \frac00$ (ind) $\lim_{x\to1}\frac{\ln x}{x-1} \xrightarrow{\text{L'Hopital}} \lim_{x\to1}\frac{\frac1x}{1} = \boxed{1}$

importante recordar

$\tan x = \frac{\operatorname{sen} x}{\cos x}$

$\operatorname{cotan} x = \frac{1}{\tan x} = \frac{\cos x}{\operatorname{sen} x}$

$\sec x = \frac{1}{\cos x}$

$\operatorname{cosec} x = \frac{1}{\operatorname{sen} x}$

$\lim\limits_{x\to 0} \dfrac{sen\ 3x}{x - \frac{3}{2} sen\ 2x}$	$\lim\limits_{x\to 0} \dfrac{sen\ 3x}{x - \frac{3}{2} sen\ 2x} = \dfrac{0}{0}$ (ind) Aplicando L'Hopital $\lim\limits_{x\to 0} \dfrac{3 \cos 3x}{1 - \frac{3}{2} \cdot 2 \cdot \cos 2x} = \boxed{\dfrac{3}{-2}}$
$\lim\limits_{x\to 0} cotan\ x\ arcsen\ x$	$\lim\limits_{x\to 0} cotan\ x \cdot arcsen\ x = \lim\limits_{x\to 0} \dfrac{\cos x \cdot arcsen\ x}{sen\ x} = \dfrac{0}{0}$ (ind) $cotan\ x = \dfrac{1}{\tan x} = \dfrac{1}{\frac{sen\ x}{\cos x}} = \dfrac{\cos x}{sen\ x}$ Aplicando L'Hopital $\lim\limits_{x\to 0} \dfrac{-sen\ x \cdot arcsen\ x + \cos x \cdot \frac{1}{\sqrt{1-x^2}}}{\cos x} = \boxed{1}$
$\lim\limits_{x\to 0} \dfrac{\ln(1+x) - sen\ x}{x\ sen\ x}$	$\lim\limits_{x\to 0} \dfrac{\ln(1+x) - sen\ x}{x \cdot sen\ x} = \dfrac{0}{0}$ (ind) Aplicando L'Hopital $\lim\limits_{x\to 0} \dfrac{\frac{1}{1+x} - \cos x}{sen\ x + x \cdot \cos x} = \dfrac{0}{0}$ (ind) Volvemos con la aplicación de L'Hopital $\lim\limits_{x\to 0} \dfrac{\frac{-1}{(1+x)^2} + sen\ x}{\cos x + \cos x - x \cdot sen\ x} = \boxed{\dfrac{-1}{2}}$

es muy importante tener un buen dominio de las derivadas