INDETERMINACIONES

CALCULO DE LIMITES INDETERMINACION $\frac{k}{0}$

- $\lim_{x\to 1}\frac{4x}{2x^2-2}$

$$\lim_{x\to 1}\frac{4x}{2x^2-2}=\frac{4\cdot 1}{2\cdot 1^2-2}=\frac{4}{0}\to\begin{cases}\lim_{x\to 1^+}\frac{4x}{2x^2-2}=\frac{4\cdot 1{,}1}{2\cdot(1{,}1)^2-2}\to\frac{+}{+}\to+\infty\\ \lim_{x\to 1^-}\frac{4x}{2x^2-2}\to\frac{4\cdot 0{,}9}{2(0.9)^2-2}\to\frac{+}{-}\to-\infty\end{cases}$$

- $\lim_{x\to -4}\frac{2x}{x^2-16}$

$$\lim_{x\to -4}\frac{2x}{x^2-16}=\frac{2\cdot(-4)}{(-4)^2-16}=\frac{-8}{0}\to\begin{cases}\lim_{x\to -4^+}\frac{2x}{x^2-16}=\frac{2\cdot(-3{,}9)}{(-3{,}9)^2-16}\to\frac{-}{-}\to+\infty\\ \lim_{x\to -4^-}\frac{2x}{x^2-16}=\frac{2\cdot(-4{,}1)}{(-4{,}1)^2-16}\to\frac{-}{+}\to-\infty\end{cases}$$

- $\lim_{x\to 2}\frac{3}{x-2}$

$$\lim_{x\to 2}\frac{3}{x-2}=\frac{3}{2-2}=\frac{3}{0}\to\begin{cases}\lim_{x\to 2^+}\frac{3}{x-2}=\frac{3}{2{,}1-2}\to\frac{+}{+}\to+\infty\\ \lim_{x\to 2^-}\frac{3}{x-2}=\frac{3}{1{,}9-2}\to\frac{+}{-}\to-\infty\end{cases}$$

- $\lim_{x\to -3}\frac{5x}{9-x^2}$

$$\lim_{x\to -3}\frac{5x}{9-x^2}=\frac{5\cdot(-3)}{9-(-3)^2}=\frac{-15}{0}\to\begin{cases}\lim_{x\to -3^+}\frac{5x}{9-x^2}=\frac{5\cdot(-2{,}9)}{9-(-2{,}9)^2}\to\frac{-}{+}\to-\infty\\ \lim_{x\to -3^-}\frac{5x}{9-x^2}=\frac{5\cdot(-3{,}1)}{9-(-3{,}1)^2}\to\frac{-}{-}\to+\infty\end{cases}$$

- $\lim_{x\to 0}\frac{2x+5}{3x}$

$$\lim_{x\to 0}\frac{2x+5}{3x}=\frac{5}{0}\to\begin{cases}\lim_{x\to 0^+}\frac{2x+5}{3x}=\frac{2\cdot(0{,}1)+5}{3\cdot 0{,}1}\to\frac{+}{+}\to+\infty\\ \lim_{x\to 0^-}\frac{2x+5}{3x}=\frac{2(-0{,}1)+5}{3\cdot(-0{,}1)}\to\frac{5}{-}\to-\infty\end{cases}$$

- $\lim_{x\to -3}\frac{x}{2x+6}$

$$\lim_{x\to -3}\frac{x}{2x+6}=\frac{-3}{0}\to\begin{cases}\lim_{x\to -3^+}\frac{x}{2x+6}=\frac{-2{,}9}{2\cdot(-2{,}9)+6}\to\frac{-}{+}\to-\infty\\ \lim_{x\to -3^-}\frac{x}{2x+6}=\frac{-3{,}1}{2\cdot(-3{,}1)+6}\to\frac{-}{-}\to+\infty\end{cases}$$

CALCULO DE LIMITES INDETERMINACION $\frac{0}{0}$

- $\lim_{x\to 4}\frac{x^2-6x+8}{x-4}$

$$\lim_{x\to 4}\frac{x^2-6x+8}{x-4}=\frac{4^2-6\cdot 4+8}{4-4}=\frac{0}{0}\ Indeterminación\ \to\frac{x^2-6x+8}{x-4}=\frac{(x-4)(x-2)}{x-4}$$

$$=(x-2)\to\lim_{x\to 4}(x-2)=4-2=2$$

- $\lim_{x\to 3}\frac{7x-21}{3-x}$

$$\lim_{x\to 3}\frac{7x-21}{3-x}=\frac{0}{0}\to Indeterminación\to\frac{7x-21}{3-x}=\frac{7(x-3)}{3-x}=\frac{-7(-x+3)}{3-x}=-7$$

$$\to\lim_{x\to 3}\ -7=-7$$

- $\lim_{x\to 0}\frac{(3+x)^2-9}{x}$

$$\lim_{x\to 0}\frac{(3+x)^2-9}{x}=\frac{0}{0}\to Indeterminación\ \to\frac{(3+x)^2-9}{x}=\frac{9+6x+x^2-9}{x}=\frac{x^2+6x}{x}$$

$$=\frac{x(x+6)}{x}=\qquad(x+6)\to\lim_{x\to 0}(x+6)=0+6=6$$

- $\lim_{x\to 2}\frac{2x^2-8}{x-2}$

$$\lim_{x\to 2}\frac{2x^2-8}{x-2}=\frac{0}{0}\to Indeterminación\to\frac{2x^2-8}{x-2}=\frac{2(x^2-4)}{x-2}=\frac{2(x+2)(x-2)}{x-2}$$

$$=2(x+2)\to$$

$$\lim_{x\to 2}2(x+2)=2(2+2)=2\cdot 4=8$$

- $\lim_{x\to 2}\frac{3x-6}{(x-2)^2}$

$$\lim_{x\to 2}\frac{3x-6}{(x-2)^2}=\frac{0}{0}\to Indeterminación\to\frac{3x-6}{(x-2)^2}=\frac{3(x-2)}{(x-2)^2}=\frac{3}{(x-2)}\to\lim_{x\to 2}\frac{3}{(x-2)}=\frac{3}{0}$$

$$\to Ind$$

$$\begin{cases}\lim_{x\to 2^+}\frac{3}{(x-2)}=\frac{3}{(2{,}1-2)}\to\frac{+}{+}\to+\infty\\ \lim_{x\to 2^-}\frac{3}{(x-2)}=\frac{3}{(1{,}9-2)}\to\frac{+}{-}\to-\infty\end{cases}$$

- $\lim_{x\to -1} \frac{x^2-1}{x^3+1}$

$$\lim_{x\to -1} \frac{x^2-1}{x^3+1} = \frac{0}{0} \to Indeterminación \to \frac{x^2-1}{x^3+1} = \frac{(x-1)(x+1)}{(x+1)(x^2-x+1)} = \frac{(x-1)}{(x^2-x+1)}$$

$$\to \lim_{x\to -1} \frac{(x-1)}{(x^2-x+1)} = \frac{-2}{3}$$

- $\lim_{x\to 0} \frac{\sqrt{1-x}-1}{x}$

$$\lim_{x\to 0} \frac{\sqrt{1-x}-1}{x} = \frac{0}{0} \to Indeterminación \to \frac{\sqrt{1-x}-1}{x} \cdot \frac{\sqrt{1-x}+1}{\sqrt{1-x}+1} = \frac{1-x-1}{x\cdot\left(\sqrt{1-x}+1\right)}$$

$$= \frac{-x}{x\left(\sqrt{1-x}+1\right)} = \frac{-1}{\sqrt{1-x}+1}$$

$$\lim_{x\to 0} \frac{-1}{\sqrt{1-x}+1} = \frac{-1}{2}$$

- $\lim_{x\to 1} \frac{x-1}{\sqrt{5-x}-2}$

$$\lim_{x\to 1} \frac{x-1}{\sqrt{5-x}-2} = \frac{0}{0} \to Indeterminación \to \frac{x-1}{\sqrt{5-x}-2} \cdot \frac{\sqrt{5-x}+2}{\sqrt{5-x}+2} = \frac{(x-1)\cdot\left(\sqrt{5-x}+2\right)}{5-x-4}$$

$$= \frac{(x-1)\cdot\left(\sqrt{5-x}+2\right)}{-x+1} = \frac{(x-1)\cdot\left(\sqrt{5-x}+2\right)}{-1(x-1)} = \frac{\left(\sqrt{5-x}+2\right)}{-1}$$

$$\to \lim_{x\to 1} \frac{\left(\sqrt{5-x}+2\right)}{-1} = -4$$

- $\lim_{x\to -3} \frac{x^3-7x+6}{x^3+3x^2+x+3}$

$$\lim_{x\to -3} \frac{x^3-7x+6}{x^3+3x^2+x+3} = \frac{(-3)^3-7(-3)+6}{(-3)^3+3(-3)^2+(-3)+3} = \frac{0}{0} \to ind$$

Tienes que descomponer tanto el numerador como denominador utilizando Ruffini.

$$x^3-7x+6=0 \to (x+3)(x-2)(x-1)$$

$$x^3+3x^2+x+3=0 \to (x+3)(x^2+1)$$

$$\lim_{x\to -3} \frac{x^3-7x+6}{x^3+3x^2+x+3} = \lim_{x\to -3} \frac{(x+3)(x-2)(x-1)}{(x+3)(x^2+1)} = \lim_{x\to -3} \frac{(x-2)(x-1)}{(x^2+1)} = \frac{20}{10} = 2$$

CALCULO DE LIMITES INDETERMINACION $\frac{\infty}{\infty}$

- $\lim_{x\to -\infty} \frac{4x^4-3x+2}{x^2+2x^4}$

$$\lim_{x\to -\infty} \frac{4x^4-3x+2}{x^2+2x^4} = \lim_{x\to \infty} \frac{4(-x)^4-3(-x)+2}{(-x)^2+2(-x)^4} = \lim_{x\to \infty} \frac{4(x)^4 3x+2}{(x)^2+2(x)^4} = \frac{\infty}{\infty} \to \frac{4x^4}{2x^4}$$

$$\to igual\ grado \to 2$$

- $\lim_{x\to \infty} \frac{(1+x)^2-1}{x}$

$$\lim_{x\to \infty} \frac{(1+x)^2-1}{x} = \frac{\infty}{\infty} \to Indeterminación \to \frac{x^2}{x} \to n > m \to +\infty$$

- $\lim_{x\to \infty} \frac{(x-1)^2}{2x^3+x}$

$$\lim_{x\to \infty} \frac{(x-1)^2}{2x^3+x} = \frac{\infty}{\infty} - Indeterminación \to \frac{x^2}{2x^3} \to n < m \to 0$$

- $\lim_{x\to \infty} \frac{3x^3-4x+2}{\sqrt{x^6+x+8}}$

$$\lim_{x\to \infty} \frac{3x^3-4x+2}{\sqrt{x^6+x+8}} = \frac{\infty}{\infty} \to\ Indeterminación \to \frac{3x^3}{\sqrt{x^6}} \to n = m \to 3$$

Cuidado con el exponente dentro de una raiz.

- $\lim_{x\to \infty} \frac{\sqrt{x^2+3}}{2x+1}$

$$¡\lim_{x\to \infty} \frac{\sqrt{x^2+3}}{2x+1} = \frac{\infty}{\infty} \to Indeterminación \to \frac{\sqrt{x^2}}{2x} \to n = m \to \frac{1}{2}$$

CALCULO DE LIMITES INDETERMINACION $\infty-\infty$

- $\lim_{x\to 1}\left(\frac{2x}{x^2-1}-\frac{1}{x-1}\right)$

$$\lim_{x\to 1}\left(\frac{2x}{x^2-1}-\frac{1}{x-1}\right)=\infty-\infty\to indeterminación\to\left(\frac{2x}{x^2-1}-\frac{1}{x-1}\right)$$

$$=\left(\frac{2x-1(x+1)}{(x-1)(x+1)}\right)=\frac{x+1}{(x-1)(x+1)}=\frac{1}{(x-1)}\to\lim_{x\to 1}\frac{1}{(x-1)}=\frac{1}{0}$$

$$\to Indeterminación\;, Limites\;laterales.$$

- $\lim_{x\to 2}\left(\frac{2}{x-2}-\frac{x+6}{x^2-4}\right)$

$$\lim_{x\to 2}\left(\frac{2}{x-2}-\frac{x+6}{x^2-4}\right)=\infty-\infty\to\;Indeterminación\;\to\frac{2}{x-2}-\frac{x+6}{x^2-4}$$

$$=\frac{2(x+2)-(x+6)}{(x+2)(x-2)}=\frac{2x+4-x-6}{(x+2)(x-2)}=\frac{x-2}{(x+2)(x-2)}=\frac{1}{(x+2)}\to$$

$$\lim_{x\to 2}\frac{1}{(x+2)}=\frac{1}{4}$$

- $\lim_{x\to\infty}\left(\sqrt{16x^2+x-5}-5x\right)$

$$\lim_{x\to\infty}\left(\sqrt{16x^2+x-5}-5x\right)=\infty-\infty\to Indeterminación\to\left(\sqrt{16x^2+x-5}-5x\right)\cdot$$

$$\frac{\sqrt{16x^2+x-5}+5x}{\sqrt{16x^2+x-5}+5x}=\frac{16x^2+x-5-25x^2}{\sqrt{16x^2+x-5}+5x}=\frac{-9x^2+x-5}{\sqrt{16x^2+x-5}+5x}=$$

$$=\frac{-\infty}{\infty}\to n>m\to-\infty$$

- $\lim_{x\to\infty}\left(\sqrt{9x^2+3x-2}-4x\right)$

$$\lim_{x\to\infty}\left(\sqrt{9x^2+3x-2}-4x\right)=\infty-\infty\to Ind$$

$$\to\frac{\left(\sqrt{9x^2+3x-2}-4x\right)\cdot\left(\sqrt{9x^2+3x-2}+4x\right)}{\left(\sqrt{9x^2+3x-2}+4x\right)}=\frac{9x^2+3x-2-16x^2}{\left(\sqrt{9x^2+3x-2}+4x\right)}$$

$$=\frac{-7x^2+3x-2}{\left(\sqrt{9x^2+3x-2}+4x\right)}\to\lim_{x\to\infty}\frac{-7x^2+3x-2}{\left(\sqrt{9x^2+3x-2}+4x\right)}=\frac{-\infty}{\infty}\to n>m\to-\infty$$

- $\lim_{x\to\infty}\left(8x-\sqrt{16x^2-3x}\right)$

$$\lim_{x\to\infty}\left(8x-\sqrt{16x^2-3x}\right)=\infty-\infty\to Ind\to\frac{\left(8x-\sqrt{16x^2-3x}\right)\cdot\left(8x+\sqrt{16x^2-3x}\right)}{\left(8x+\sqrt{16x^2-3x}\right)}$$

$$=\frac{64x^2-(16x^2-3x)}{\left(8x+\sqrt{16x^2-3x}\right)}=\frac{48x^2+3x}{\left(8x+\sqrt{16x^2-3x}\right)}\to\lim_{x\to\infty}\frac{48x^2+3x}{\left(8x+\sqrt{16x^2-3x}\right)}=\frac{\infty}{\infty}$$

$$\to n>m\to\infty$$

- $\lim_{x\to\infty}\left(\sqrt{x^4+6x}-\sqrt{x^4+8}\right)$

$$\lim_{x\to\infty}\left(\sqrt{x^4+6x}-\sqrt{x^4+8}\right)=\infty-\infty\to Ind\to\frac{\left(\sqrt{x^4+6x}-\sqrt{x^4+8}\right)\cdot\left(\sqrt{x^4+6x}+\sqrt{x^4+8}\right)}{\left(\sqrt{x^4+6x}+\sqrt{x^4+8}\right)}$$

$$=\frac{x^4+6x-x^4-8}{\left(\sqrt{x^4+6x}+\sqrt{x^4+8}\right)}=\frac{6x-8}{\left(\sqrt{x^4+6x}+\sqrt{x^4+8}\right)}$$

$$\to\lim_{x\to\infty}\frac{6x-8}{\left(\sqrt{x^4+6x}+\sqrt{x^4+8}\right)}=\frac{\infty}{\infty}\to n<m\to 0$$

CALCULO DE LIMITES INDETERMINACION 1^{∞}

- $\lim_{x\to\infty}\left(\frac{4x+3}{4x-5}\right)^{2x}$

$$\lim_{x\to\infty}\left(\frac{4x+3}{4x-5}\right)^{2x} = \left(\frac{\infty}{\infty}\right)^{\infty} \to 1^{\infty} \to Indeterminación \to e^{\lim_{x\to\infty} 2x\cdot\left[\left(\frac{4x+3}{4x-5}\right)-1\right]} = e^{4}$$

$$\lim_{x\to\infty} 2x\cdot\left[\left(\frac{4x+3}{4x-5}\right)-1\right] = \lim_{x\to\infty} 2x\left[\frac{4x+3-(4x-5)}{4x-5}\right] = \lim_{x\to\infty} 2x\left[\frac{8}{4x-5}\right] = \lim_{x\to\infty}\frac{16x}{4x-5} = \frac{\infty}{\infty}$$

$$\to \quad n = m \to \frac{16}{4} = 4$$

- $\lim_{x\to\infty}\left(\frac{x^2}{x^2+2}\right)^{x}$

$$\lim_{x\to\infty}\left(\frac{x^2}{x^2+2}\right)^{x} = 1^{\infty} \to Indeterminación \to e^{\lim_{x\to\infty} x\cdot\left(\frac{x^2}{x^2+2}-1\right)} = e^{0} = 1$$

$$\lim_{x\to\infty} x\cdot\left(\frac{x^2}{x^2+2}-1\right) = \lim_{x\to\infty} x\cdot\left[\frac{x^2-(x^2+2)}{x^2+2}\right] = \lim_{x\to\infty} x\cdot\frac{-2}{x^2+2} = \lim_{x\to\infty}\frac{-2x}{x^2+2} = \frac{-\infty}{\infty}$$

- $\lim_{x\to 0}(1+4x)^{\frac{3}{x}}$

$$\lim_{x\to 0}(1+4x)^{\frac{3}{x}} = (1+4\cdot 0)^{\frac{3}{0}} \to 1^{\infty} \to Ind \to e^{\lim_{x\to 0}\frac{3}{x}\cdot[1+4x-1]} = e^{12}$$

$$\lim_{x\to 0}\frac{3}{x}\cdot[1+4x-1] = \lim_{x\to 0}\frac{3}{x}\cdot 4x = \lim_{x\to 0}\frac{12x}{x} = \frac{0}{0} \to Ind \to \frac{12x}{x} = 12 \to \lim_{x\to 0} 12 = 12$$

- $\lim_{x\to\infty}\left(1-\frac{3}{x}\right)^{x}$

$$\lim_{x\to\infty}\left(1-\frac{3}{x}\right)^{x} = \left(1-\frac{3}{\infty}\right)^{\infty} = 1^{\infty} \to Ind \to e^{\lim_{x\to\infty} x\cdot\left(1-\frac{3}{x}-1\right)} = e^{-3} = \frac{1}{e^{3}}$$

$$\lim_{x\to\infty} x\cdot\left(1-\frac{3}{x}-1\right) = \lim_{x\to\infty}\frac{-3x}{x} = \lim_{x\to\infty} -3 = -3$$

- $\lim_{x\to\infty}\left(\frac{3x^2}{3x^2-7}\right)^{6x}$

$$\lim_{x\to\infty}\left(\frac{3x^2}{3x^2-7}\right)^{6x} = \left(\frac{\infty}{\infty}\right)^{\infty} \to 1^{\infty} \to Ind \to e^{\lim_{x\to\infty} 6x\cdot\left(\frac{3x^2}{3x^2-7}-1\right)} = e^{0}$$

$$\lim_{x\to\infty} 6x\cdot\left(\frac{3x^2}{3x^2-7}-1\right) = \lim_{x\to\infty} 6x\cdot\left[\frac{3x^2-(3x^2-7)}{3x^2-7}\right] = \lim_{x\to\infty} 6x\cdot\left[\frac{-7}{3x^2-7}\right] = \lim_{x\to\infty}\frac{-42x}{3x^2-7} = \frac{\infty}{\infty}$$

$$\to n < m \to 0$$

- $\lim_{x\to-\infty}\left(1-\frac{2x}{x^2-1}\right)^{-4x}$

$$\lim_{x\to-\infty}\left(1-\frac{2x}{x^2-1}\right)^{-4x} = 1^\infty \to IND \to e^{\lim_{x\to-\infty} -4x\left[1-\frac{2x}{x^2-1}-1\right]} = e^{\lim_{x\to-\infty} -4x\left[-\frac{2x}{x^2-1}\right]}$$

$$= e^{\lim_{x\to-\infty}\left[\frac{8x^2}{x^2-1}\right]} = e^8$$

$$\lim_{x\to-\infty}\frac{8x^2}{x^2-1} \to \lim_{x\to-\infty}\frac{8x^2}{x^2} = 8$$