$A_1$

$$\begin{cases} x + y + \alpha z = \alpha \\ 2x + \alpha y + \alpha z = 1 \\ x + \alpha y + z = 1 \end{cases} \rightsquigarrow \left(\begin{array}{ccc|c} 1 & 1 & \alpha & \alpha \\ 2 & \alpha & \alpha & 1 \\ 1 & \alpha & 1 & 1 \end{array}\right)$$

$$|A| = \alpha + \underline{2\alpha^2} + \alpha \underline{-\alpha^2} - 2 \underline{-\alpha^2} = 2\alpha - 2$$

$$2\alpha - 2 = 0 \rightarrow \alpha = 1$$



- $\alpha \neq 1 \rightarrow |A| \neq 0 \rightarrow$ Rango $(A) = 3$
  Rang $(A^*) = 3 \rightsquigarrow$ S.C.D
  $n =$ incognitas $= 3$

- $\alpha = 1 \rightarrow |A| = 0 \rightarrow$ Rang $(A) \leq 2$

$$\begin{vmatrix} 1 & 1 \\ 2 & 1 \end{vmatrix} = 1 - 2 = -1 \neq 0 \quad \text{Rango } (A) = 2$$

$\rightsquigarrow$ S.C.I

$$|A^*| = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 1 & 1 \\ 1 & 1 & 1 \end{vmatrix} = 0 \rightarrow \text{Rango } (A^*) = 2$$

Para $\alpha = -1 \rightarrow$ S.C.D $\rightsquigarrow$ CRAMER

$$x = \frac{\begin{vmatrix} -1 & 1 & -1 \\ 1 & -1 & -1 \\ 1 & -1 & 1 \end{vmatrix}}{-4} = \frac{1 - 1 + 1 - 1 - 1 + 1}{-4} = 0$$

$$y = \frac{\begin{vmatrix} 1 & -1 & -1 \\ 2 & 1 & -1 \\ 1 & 1 & 1 \end{vmatrix}}{-4} = \frac{1 - 2 + 1 + 1 + 2 + 1}{-4} = -1 \rightsquigarrow \begin{matrix} (x, y, z) \\ (0, -1, 0) \end{matrix}$$

$$z = \frac{\begin{vmatrix} 1 & 1 & -1 \\ 2 & -1 & 1 \\ 1 & -1 & 1 \end{vmatrix}}{-4} = \frac{-1+2+1-1-2+1}{-4} = 0$$

Para $\alpha = 1 \rightsquigarrow$ S.C.I

$$\begin{cases} x + y + z = 1 \\ 2x + y + z = 1 \\ x + y + z = 1 \end{cases} \rightsquigarrow \begin{matrix} x + y + z = 1 \\ 2x + y + z = 1 \\ \boxed{z = t} \end{matrix} \rightsquigarrow \begin{matrix} x + y = 1 - t \\ 2x + y = 1 - t \\ \hline -x = 0 \\ \boxed{x = 0} \end{matrix}$$

$$x + y + z = 1 \longrightarrow \boxed{y = 1 - t}$$

---

B1

$$A = \begin{pmatrix} m & m & 2 \\ 1 & m-2 & 0 \\ 0 & 2 & 2 \end{pmatrix} \qquad m = 0$$

$$|A| = 2m(m-2) + 4 - 2m = 2m^2 - 4m - 2m + 4 = 2m^2 - 6m + 4$$

$$2m^2 - 6m + 4 = 0$$

$$m = \frac{6 \pm \sqrt{36 - 4(2)(4)}}{4} \longrightarrow m = \begin{cases} \frac{6+2}{4} = 2 \\ \frac{6-2}{4} = 1 \end{cases}$$

Para $m = 2$ y $m = 1 \rightsquigarrow$ No tiene Inversa

Para $m \neq 2$ y $m \neq 1 \longrightarrow$ si " "

$\boxed{m=0}$

$$A=\begin{pmatrix}0&0&2\\1&-2&0\\0&2&2\end{pmatrix} \qquad |A|=4$$

$$A_{adj}=\begin{pmatrix}-4&-2&2\\4&0&0\\4&2&0\end{pmatrix} \rightarrow (A_{adj})^t=\begin{pmatrix}-4&4&4\\-2&0&2\\2&0&0\end{pmatrix}$$

$$A^{-1}=\frac{1}{|A|}\cdot(A_{adj})^t \rightarrow A^{-1}=\frac{1}{4}\begin{pmatrix}-4&4&4\\-2&0&2\\2&0&0\end{pmatrix}$$

## A2

$$r\equiv\begin{cases}x=t\\y=2t\\z=0\end{cases} \rightsquigarrow (x,y,z)=(t,2t,0) \qquad Q(0,0,0)$$

$$\pi = x+y+z-2=0$$

distancia $\overrightarrow{PQ}$

$$\overrightarrow{QP}=(t,2t,0)$$

$$|\overrightarrow{QP}|=\sqrt{t^2+4t^2}=\sqrt{5t^2}$$

distancia punto P plano

$$dist(P,\pi)=\frac{|t+2t-2|}{\sqrt{3}}$$

$$\sqrt{5t^2}=\frac{3t-2}{\sqrt{3}} \rightsquigarrow \sqrt{15t^2}=3t-2$$

$$15t^2=9t^2-12t+4 \rightsquigarrow 6t^2+12t+4=0$$

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$$t = \frac{-12 \pm \sqrt{144 - 4(6)(4)}}{12} \rightarrow t = <$$

Tenemos dos puntos de r que estan a la misma distancia del $(0,0,0)$ y del plano.

B2

$P(1,2,\alpha)$

$\pi = x + y + 2z = 3$

recta con $\begin{array}{l} \vec{n} = (1,1,2) \\ P(1,2,\alpha) \end{array} \rightsquigarrow r = \begin{cases} x = 1 + t \\ y = 2 + t \\ z = \alpha + 2t \end{cases}$

Punto de corte entre recta y plano

$1 + t + 2 + t + 2(\alpha + 2t) = 3$

$1 + t + 2 + t + 2\alpha + 4t = 3$

$6t = -2\alpha$

$\boxed{t = \frac{-\alpha}{3}}$

punto Medio

$M \begin{cases} x = \frac{3-\alpha}{3} \\ y = \frac{6-\alpha}{3} \\ z = \frac{\alpha}{3} \end{cases}$

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Simetrico $= 2M - P$

$2\left(\frac{3-\alpha}{3}, \frac{6-\alpha}{3}, \frac{\alpha}{3}\right) - (1,2,\alpha) =$

$$= \left( \frac{6-2\alpha}{3} - 1 \ , \ \frac{12-2\alpha}{3} - 2 \ , \ \frac{2\alpha}{3} - \alpha \right)$$

$$= \left( \frac{3-2\alpha}{3} \ , \ \frac{6-2\alpha}{3} \ , \ \frac{-\alpha}{3} \right)$$

**A3**

$$f(x) = (x-1)^2 \cdot e^{-2x}$$

$$f'(x) = 2(x-1) \cdot e^{-2x} + (x-1)^2 \cdot (-2) \cdot e^{-2x}$$

$$f'(x) = e^{-2x} \left( 2x - 2 + (x-1)^2 (-2) \right)$$

$$f'(x) = e^{-2x} \left( 2x - 2 - 2x^2 + 4x - 2 \right)$$

$$f'(x) = e^{-2x} \left( -2x^2 + 6x - 4 \right)$$

$$f'(x) = 0 \rightsquigarrow \begin{cases} e^{-2x} \neq 0 \ \nexists \\ -2x^2 + 6x - 4 = 0 \rightarrow \end{cases}$$

$$x = \frac{-6 \pm \sqrt{36 - 4(-2)(-4)}}{-4} \rightarrow x = \begin{cases} 1 \\ 2 \end{cases}$$

| | | min | | MAX | |
|---|---|---|---|---|---|
| $f(x) \rightarrow$ | $\searrow$ | | $\nearrow$ | | $\searrow$ |
| | (0) | 1 | (1'5) | 2 | (3) |
| $f'(x) \rightarrow$ | $-$ | | $+$ | | $-$ |

B3

$f(x) = x^3 + Ax^2 + Bx + C \rightsquigarrow f'(x) = 3x^2 + 2Ax + B$

$(1,0) \rightsquigarrow 1 + A + B + C = 0$

$f'(-1) = 2 \rightarrow 3 - 2A + B = 2$

$f'(3) = 2 \rightsquigarrow 27 + 6A + B = 2$

$$\rightsquigarrow \begin{array}{r} -2A + B = -1 \\ +6A + B = -25 \\ \hline -8A \quad = 24 \end{array}$$

$\boxed{A = -3}$

$-2A + B = -1$

$6 + B = -1$

$\boxed{B = -7}$

$A + B + C = -1$

$-3 - 7 + C = -1$

$\boxed{C = 9}$

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A4

$$\int \frac{7x+13}{(x+1)(x^2-x-2)}\,dx = \int \frac{A}{(x-2)}\,dx + \int \frac{B}{(x+1)}\,dx + \int \frac{C}{(x+1)^2}$$

$$(x+1)(x^2-x-2) = 0 \begin{cases} x = -1 \\ x^2 - x - 2 = 0 \longrightarrow \end{cases}$$

$$x = \frac{1 \pm \sqrt{1-4(1)(-2)}}{2} \longrightarrow x = \begin{cases} 2 \\ -1 \end{cases}$$

$7x + 13 = A(x+1)^2 + B(x+1)(x-2) + C(x-2)$

- $x = -1 \longrightarrow 6 = -3C \longrightarrow \boxed{C = -2}$

• $X = 2 \longrightarrow 27 = 9A \rightsquigarrow A = \frac{27}{9} \rightsquigarrow \boxed{A = 3}$

• $X = 0 \longrightarrow 13 = A - 2B - 2C$

$13 = 3 - 2B + 4 \longrightarrow 6 = -2B \longrightarrow \boxed{B = -3}$

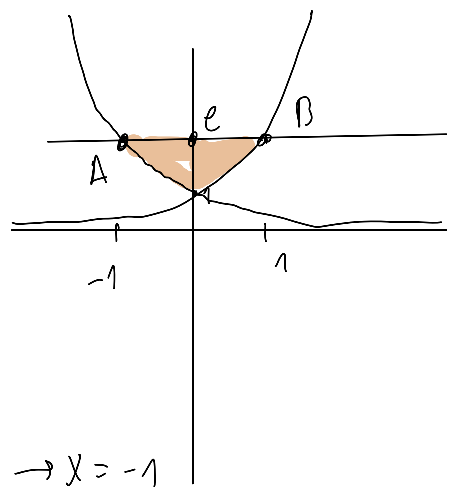
$= 3\ln|x-2| - 3\ln|x+1| + \frac{2}{(x+1)} + k.$

B4

$f(x) = e^{x}$

$g(x) = e^{-x}$

$y = e$






puntos A y B

$A \to e^{-x} = e \to -x = 1 \to x = -1$

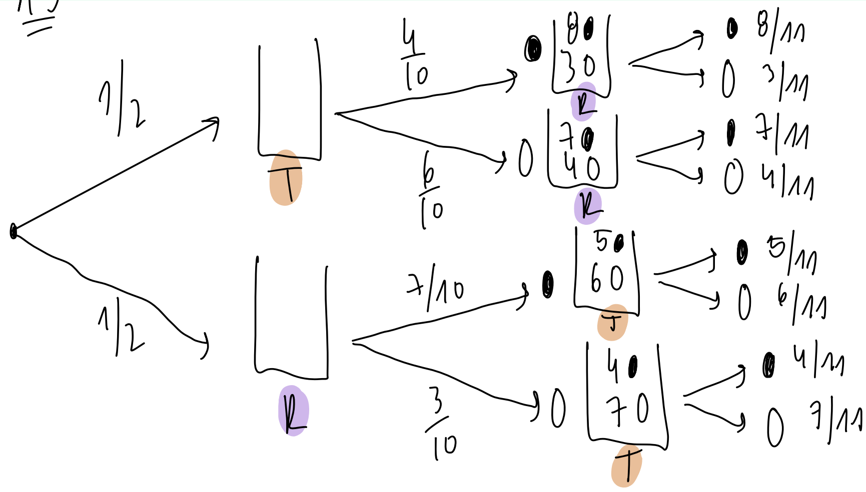
$B \to e^{x} = e \to x = 1$

$$\int_{-1}^{0} e - e^{-x}\,dx + \int_{0}^{1} e - e^{x}\,dx =$$

$$\left. ex + e^{-x} \right]_{-1}^{0} + \left. ex - e^{x} \right]_{0}^{1} =$$

$$(1) - (-e + e) + (0 - (-1)) = 2\ u^2$$

## A5

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$$P(\text{negras las dos}) = \frac{1}{2}\cdot\frac{4}{10}\cdot\frac{8}{11} + \frac{1}{2}\cdot\frac{7}{10}\cdot\frac{5}{11}$$

$$P(\text{blancas las dos}) = \frac{1}{2}\cdot\frac{6}{10}\cdot\frac{7}{11} + \frac{1}{2}\cdot\frac{3}{10}\cdot\frac{7}{11}$$

$$P(\text{distinto color}) = 1 - p(\text{negras las 2}) - p(\text{blancas las dos})$$

## B5

$N(52,\ 6'5)$

$$P(50 < x < 68) = P\left(\frac{50-52}{6'5} < z < \frac{68-52}{6'5}\right)$$

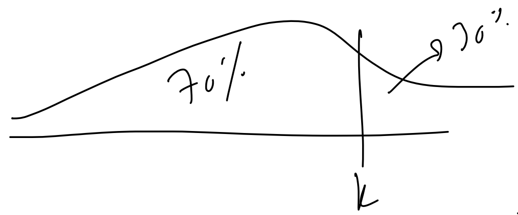
$$P(-0'31 < z < 2'46)$$

$$P(z < 2'46) - P(z < -0'31)$$

$$P(z < 2'46) - 1 + P(z < 0'31)$$

$$0'9931 - 1 + 0'6217 = 0'6148$$

$61'48\%$




$$P(x > k) = 0'3$$

$$P(x < k) = 0'7$$

$$\frac{k-52}{6'5} = 0'525 \rightarrow k = 55'41$$